GEOMETRY.

135. Proposed by WILLIAM HOOVER, A.M., Ph.D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

If a hyperbola be described touching the four sides of a quadrilateral which is inscribed in a circle, and one focus lie on the circle, the other focus will also lie on the circle.

Solution by the PROPOSER.

Using quadrilinear notation, the equation to the circle circumscribing the quadrilateral whose sides are given by $\alpha=0$, $\beta=0$, $\gamma=0$, $\delta=0$, is $\alpha\gamma=\beta\delta$....(1).

Now, it is well known that if the coördinates of one focus of a conic tangent to a given line be α', β', γ', δ', those of the other focus are proportional to $1/\alpha'$, $1/\beta'$, $1/\gamma'$, $1/\delta'$.

But by the problem, α', β', γ', δ' is on (1); then $\alpha'\gamma'=\beta'\delta'$....(2), or

$$\frac{1}{\alpha'\gamma'}=\frac{1}{\beta'\delta'}\text{....(3)}.$$

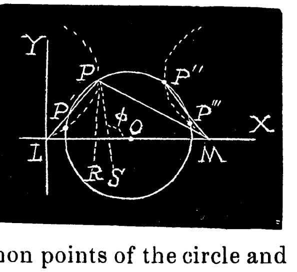
Substituting the reciprocals in (1) gives (3) also, and proves the theorem.

136. Proposed by J. OWEN MAHONEY, B. E., M. Sc., Professor of Mathematics, Central High School, Dallas, Tex.

Construct a triangle having given the base, the median line to the base, and the difference of the base angles.

I. Solution by B. L. REMICK, Instructor of Mathematics, Bradley Institute, Peoria, Ill.

Let $LM=a=$the base, $CP=m=$median to the base, $\alpha-\beta=$difference of base angles.



Then vertex P of required triangle lies on circle about C (mid point of LM) as center with radius m; it also lies on the locus of point of intersection of straight lines through L, M forming angles with base having the required constant difference. We propose to show that this latter locus is an equilateral hyperbola and that our problem has therefore four solutions corresponding to the four common points of the circle and hyperbola.

$\angle RPS$ between the perpendicular and angle bisector$=\frac{1}{2}(\alpha-\beta)$ by a well known result in geometry; and hence we have to consider the locus of intersection of two straight lines passing through two given points L, M so that the angle bisector remains parallel to itself.

Let coördinates of P be (x_1, y_1).

Equation PL is $y_1x-x_1y=0$.

Equation PM is $y_1x+(a-x_1)y-ay_1=0$.